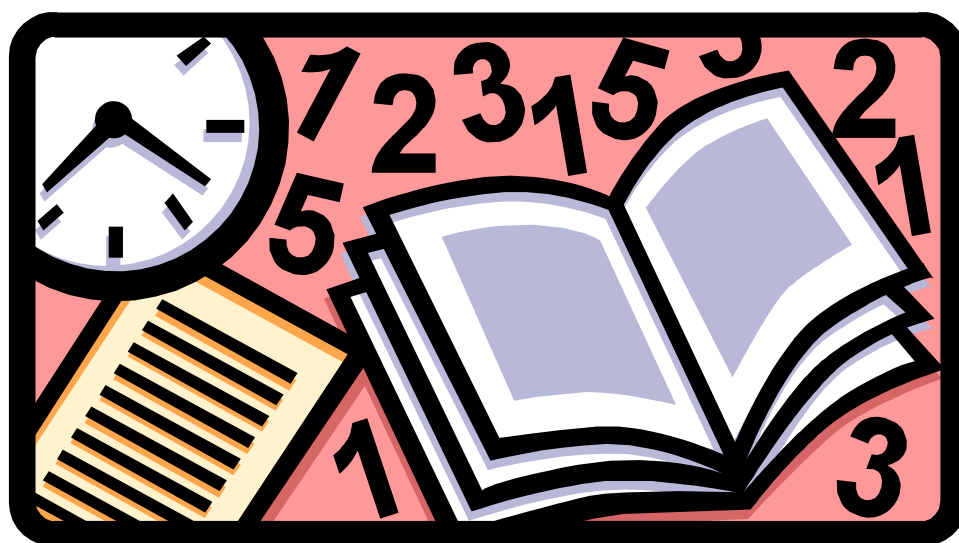


Strathblane Primary School

Numeracy Support Booklet



Written and Mental Strategies

Numeracy Development Group 2017

Introduction

What is the purpose of the booklet?

This booklet has been produced to give guidance to pupils and parents on how certain common Numeracy topics are taught in mathematics and throughout the school. All staff have been issued with a copy of the booklet. It is hoped that using a consistent approach across all subjects will make it easier for pupils to progress.

How can it be used?

If you are helping your child with their homework, you can refer to the booklet to see what methods are being taught in school. Look up the relevant page for a step by step guide. Pupils can also use the booklet in school or at home to help with numeracy work.

If you would like to know what your child is studying in mathematics, please check the website, your child's homework diary or ask their teacher.

Why do some topics include more than one method?

There is not one set way to solve maths problems and children should be encouraged to use a method to suit their learning style or personal preference.

In some cases (e.g. percentages), the method used will be dependent on the level of difficulty of the question, and whether or not a calculator is permitted.

For mental calculations, pupils should be encouraged to develop a variety of strategies so that they can select the most appropriate method in any given situation.

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Addition

Mental strategies



There are a number of useful mental strategies for addition. Some examples are given below.

Example Calculate $54 + 27$

Method 1 Add tens, then add units, then add together

$$50 + 20 = 70 \qquad 4 + 7 = 11 \qquad 70 + 11 = 81$$

Method 2 Split up number to be added into tens and units and add separately.

$$54 + 20 = 74 \qquad 74 + 7 = 81$$

Method 3 Round up to nearest 10, then subtract

$$54 + 30 = 84 \quad \text{but } 30 \text{ is } 3 \text{ too much so subtract } 3;$$
$$84 - 3 = 81$$

Written Method

When adding numbers, ensure that the numbers are lined up according to **place value** (e.g ThHTU). Start at right hand side, write down units, carry tens.

Example Add 3032 and 589

ThHTU				
3032		3032		3032
+589		+589		+589
-----		-----		-----
1	→	21	→	621
-----		-----		-----
1		11		11
-----		-----		-----
3621				3621

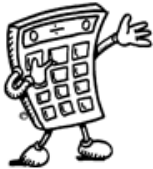
$2 + 9 = 11$

$3 + 8 + 1 = 12$

$0 + 5 + 1 = 6$

$3 + 0 = 3$

Subtraction



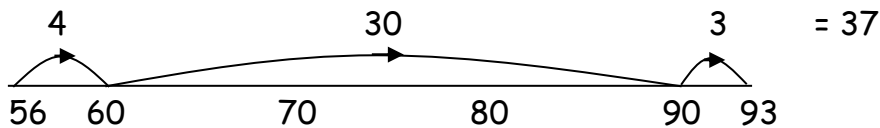
We use decomposition as a written method for subtraction (see below). Alternative methods may be used for mental calculations.

Mental Strategies

Example Calculate $93 - 56$

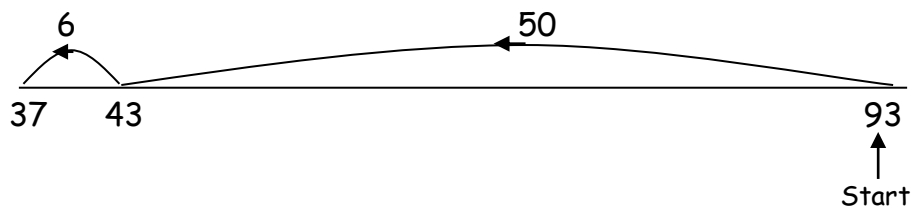
Method 1 Count on

Count on from 56 until you reach 93. This can be done in several ways e.g.



Method 2 Break up the number being subtracted

e.g. subtract 50, then subtract 6 $93 - 50 = 43$
 $43 - 6 = 37$



Written Method

Example 1 $4590 - 386$

$$\begin{array}{r} 81 \\ 4590 \\ - 386 \\ \hline 4204 \end{array}$$

Example 2 Subtract 692 from 14597

We do not
"borrow and
pay back".

$$\begin{array}{r} 31 \\ 14597 \\ - 692 \\ \hline 13905 \end{array}$$

Multiplication 1



It is essential that you know all of the multiplication tables from 1 to 10. These are shown in the tables square below.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Mental Strategies

Example Find 39×6

Method 1

$$\begin{array}{l} 30 \times 6 \\ = 180 \end{array}$$

$$\begin{array}{l} 9 \times 6 \\ = 54 \end{array}$$

$$\begin{array}{l} 180 + 54 \\ = 234 \end{array}$$

Method 2

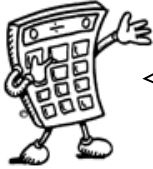
$$\begin{array}{l} 40 \times 6 \\ = 240 \end{array}$$

40 is 1 group of 6 too many so take away 1×6

$$\begin{array}{l} 240 - 6 \\ = 234 \end{array}$$

Multiplication 2

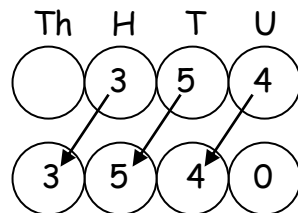
Multiplying by multiples of 10 and 100



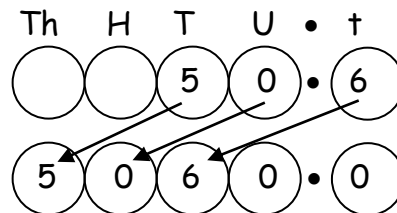
To multiply by **10** you move every digit *one* place to the left.

To multiply by **100** you move every digit *two* places to the left.

Example 1 (a) Multiply 354 by 10 (b) Multiply 50.6 by 100



$$354 \times 10 = 3540$$



$$50.6 \times 100 = 5060$$

(c) 35×30

To multiply by 30,
multiply by 3,
then by 10.

$$\begin{aligned} 35 \times 3 &= 105 \\ 105 \times 10 &= 1050 \end{aligned}$$

$$\text{so } 35 \times 30 = 1050$$

(d) 436×600

To multiply by
600, multiply by 6,
then by 100.

$$\begin{aligned} 436 \times 6 &= 2616 \\ 2616 \times 100 &= 261600 \end{aligned}$$

$$\text{so } 436 \times 600 = 261600$$

Multiplication 3

Written method for multiplication



$$\begin{array}{r} 694 \\ \times \quad 4 \\ \hline 2776 \\ \hline 231 \end{array}$$

Written method for long multiplication

$$\begin{array}{r} 157 \\ \times \quad 93 \\ \hline 471 \\ 14130 \\ \hline 14601 \\ \hline 1 \end{array}$$

Doubling and halving strategies

If you are multiplying two digit numbers, you can half one of the numbers and double the other. It will give you the same answer.

$$18 \times 25 =$$

$$9 \times 50 =$$

$$450$$

$$24 \times 15 =$$

$$12 \times 30 =$$

$$360$$

Division

Written division



Written Method

Example 1 There are 192 pupils in first year, shared equally between 8 classes. How many pupils are in each class?

$$\begin{array}{r} 024 \\ 8 \overline{)1932} \end{array}$$

There are 24 pupils in each class

Example 2 Divide 4.74 by 3

$$\begin{array}{r} 1.58 \\ 3 \overline{)4.724} \end{array}$$

When dividing a decimal number by a whole number, the decimal points must stay in line.

Example 3 A jug contains 2.2 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

$$\begin{array}{r} 0.275 \\ 8 \overline{)2.26040} \end{array}$$

Each glass contains 0.275 litres

If you have a remainder at the end of a calculation, add a zero onto the end of the decimal and continue with the calculation.

Decimals

Working with Decimals

$29.1 + 6 + 104.42$

$30.6 - 9.58$

53.28×9



Written Method

$$\begin{array}{r} \text{HTU . t h} \\ 29.1 \\ 6 \\ + 104.42 \\ \hline 139.52 \\ \hline 1 \end{array}$$

$$\begin{array}{r} \overset{2}{\cancel{3}}\overset{1}{0}.\overset{5}{\cancel{6}}\overset{1}{0} \\ - 9.58 \\ \hline 21.02 \end{array}$$

$$\begin{array}{r} 53.28 \\ \times 9 \\ \hline 479.52 \\ \hline 227 \end{array}$$

Multiplying and Dividing with Decimals

Place Value

Place value defines our counting system.

Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths
6	1	5	.	8	3

This is the number six hundred and fifteen point eight three.

Note that the decimal point is located between the units and tenths columns and does not move.

Multiplication by 10, 100 and 1000

For multiplication by 10, 100 and 1000 the digits move to the left by 1, 2 and 3 places, respectively.

Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths
			5	.	8	3
		5	8	.	3	
	5	8	3			
5	8	3	0			

($\times 10$)
 ($\times 100$)
 ($\times 1000$)

Division by 10, 100 and 1000

For division by 10, 100 and 1000 the digits move to the right by 1, 2 and 3 places, respectively.

Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
5	8	3				
	5	8	.	3		
		5	.	8	3	
		0	.	5	8	3

($\div 10$)
 ($\div 100$)
 ($\div 1000$)

The pattern continues in this way for powers of 10 larger than 1000. Moreover, we do not say “add a zero” for multiplication and “take off a zero” for division as this can cause significant problems when decimals are involved.

Order of Calculation (BODMAS)

Consider this: What is the answer to $2 + 5 \times 8$?

Is it $7 \times 8 = 56$ or $2 + 40 = 42$?

The correct answer is 42.



Calculations which have more than one operation need to be done in a particular order. The order can be remembered by using the mnemonic **BODMAS**

The **BODMAS** rule tells us which operations should be done first.

BODMAS represents:

(B)rackets

(O)ther operations (e.g fractions, squared)

(D)ivide

(M)ultiply

(A)dd

(S)ubtract

Scientific calculators use this rule, some basic calculators may not, so take care in their use.

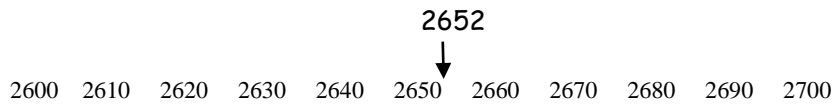
Example 1 $15 - 12 \div 6$ BODMAS tells us to divide first
 = $15 - 2$
 = 13

Example 2 $(9 + 5) \times 6$ BODMAS tells us to work out the
 = 14×6 brackets first
 = 84

Example 3 $18 + 6 \div (5-2)$ Brackets first
 = $18 + 6 \div 3$ Then divide
 = $18 + 2$ Now add
 = 20

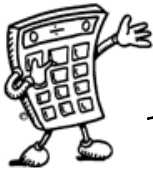
Rounding

Numbers can be rounded to give an approximation.



2652 rounded to the nearest 10 is 2650.

2652 rounded to the nearest 100 is 2700.



When rounding numbers which are exactly in the middle, convention is to **round up**.

7865 rounded to the nearest 10 is 7870.

The same principle applies to rounding decimal numbers.

In general, to round a number, we must first identify the place value to which we want to round. We must then look at the next digit to the right (the "check digit") - if it is 5 or more round up.

Example 1 Round 46,753 to the nearest thousand.

6 is the digit in the thousands column - the check digit (in the hundreds column) is a 7, so round up.

$$\begin{array}{l} \underline{46} \ 753 \\ = 47 \ 000 \text{ to the nearest thousand} \end{array}$$

Example 2 Round 1.57359 to 2 decimal places

The second number after the decimal point is a 7 - the check digit (the third number after the decimal point) is a 3, so round down.

$$\begin{array}{l} 1.\underline{57}359 \\ = 1.57 \text{ to 2 decimal places} \end{array}$$

Estimation



We can use rounded numbers to give us an approximate answer to a calculation. This allows us to check that our answer is sensible.

Example 1

Tickets for a concert were sold over 4 days. The number of tickets sold each day was recorded in the table below. How many tickets were sold in total?

Monday	Tuesday	Wednesday	Thursday
486	205	197	321

$$\text{Estimate} = 500 + 200 + 200 + 300 = 1200$$

Calculate:

$$\begin{array}{r} 486 \\ 205 \\ 197 \\ +321 \\ \hline 1209 \end{array} \quad \text{Answer} = 1209 \text{ tickets}$$

Example 2

A bar of chocolate weighs 42g. There are 48 bars of chocolate in a box. What is the total weight of chocolate in the box?

$$\text{Estimate} = 50 \times 40 = 2000\text{g}$$

Calculate:

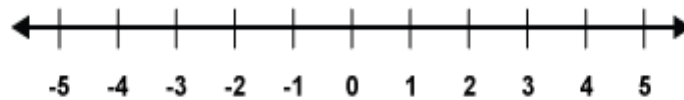
$$\begin{array}{r} 42 \\ \times 48 \\ \hline 336 \\ 1680 \\ \hline 2016 \end{array} \quad \text{Answer} = 2016\text{g}$$

Integers

Integers



Integers are whole numbers which can be either positive or negative. Negative numbers are below zero and written with '-'. Integers are usually taught in the context of temperature or profit and loss.



Adding and subtracting integers

Examples

- a) $-4 + 3 = -1$ (start at -4 and move right 3 places)
- b) $-8 + 10 = 2$ (start at -8 and move right 10 places)
- c) $-5 - 2 = -7$ (start at -5 and move left 2 places)
- d) $-11 - 7 = -18$ (start at -11 and move left 7 places)

Time 1

12-hour clock

Time can be displayed on a clock face, or digital clock.



05:15

12 hour clock

When using the 12 hour clock:-

- a.m. is from midnight to noon
e.g. 6.03am is 3 minutes past 6 in the morning
- p.m. is from noon to midnight.
e.g. 6.03pm is 3 minutes past 6 in the evening

These clocks both show fifteen minutes past five, or quarter past five.

24 hour clock

Times written using the 24 hour clock require **4 digits** ranging from 00:00 to 23:59.

- Midnight is written as 00:00;
- 12 noon is written as 12:00 as 12 hours have passed to reach midday
- The hours after 12:00 (noon) are 13:00, 14:00, 15:00, etc until midnight (00.00).
- Remember, when we change from 23:59 (1 minute to midnight), the clock changes to 00:00 (**not 24:00**) and a new day begins!
- We do not use a.m / p.m with the 24 hour clock because we already know whether it's am/pm depending on how it is written

Examples

01:00 = 1.00am

13:00 = 1.00pm

11:59 = 11.59am

23:59 = 11.59pm

12:00 = 12.00pm (noon)

00:00 = 12.00am (midnight)

Time 2



It is essential to know the number of months, weeks and days in a year, and the number of days in each month.

Time Facts

In 1 year, there are: 365 days (366 in a leap year)
 52 weeks
 12 months

The number of days in each month can be remembered using the rhyme:

"30 days has September,
April, June and November,
All the rest have 31,
Except February alone,
Which has 28 days clear,
And 29 in each leap year."

Periods of Time

1000	years	=	1 millennium
100	years	=	1 century
10	years	=	1 decade
60	seconds	=	1 minute
60	minutes	=	1 hour
24	hours	=	1 day
7	days	=	1 week
12	months	=	1 year
52	weeks	=	1 year
365	days	=	1 year
366	days	=	1 leap year

Time 3

Time Intervals (in hours and minutes)

Short Time Intervals

Example - How long is it from 2.45pm to 6.05pm?

There are different ways of working out the duration. An effective method is to use the 'counting on' strategy.

Step 1 - Create a timeline showing the start time (e.g 2.45pm),

Step 2 - Now write down the next 'whole hour' (e.g. 3.00pm)

Step 3 - Now put the 'whole hour that comes becomes your finish time (e.g 6.00pm)

Step 4 - Put the finish time on your timeline (e.g 6.05pm)

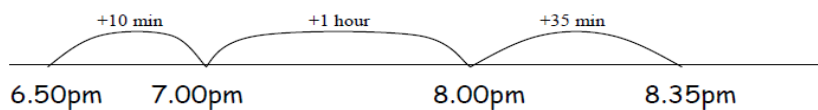
Step 5 - Now calculate the duration of time in jumps as shown below

The length of a time interval can be found by counting on.

Example

A film starts at 6.50pm and ends at 8.35pm.

What is the length of the film.



Length of film = 10minutes + 1 hour + 35 minutes = 1 hour 45 minutes

Do not use subtraction.

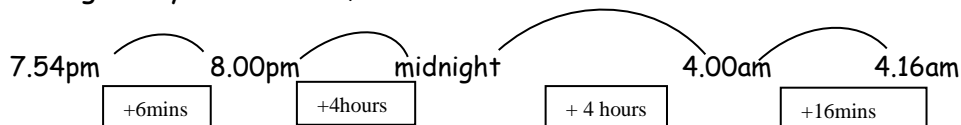


Step 6 - Finally add up your hours and minutes (3 hours, plus 15 minutes, plus 5 minutes = 3 hours and 20 minutes)

Longer Time Intervals

Example - How long is it from 7.54pm to 4.16am?

Use the same method above but to make the calculation easier, include midnight in your timeline, as shown below



In total we have 8 hours and 22 minutes.

Time 4

Reading Timetables

Timetables are presented in many ways. You will need to read information in rows (going across) or in columns (going down).

Look at the following timetable:-

Daily Train Times Glasgow-Crianlarich				
Glasgow	08.20	08.50	12.40	18.20
Dalmuir	08.38	09.08	12.58	18.38
Tarbet	09.35	10.05	13.55	19.35
Ardlui	09.51	10.21	14.11	19.51
Crianlarich Campsite	10.05	10.35	14.25	20.05

- We can tell it is in 24 hour time
- Look at the first row. We can see the 4 departure times from Glasgow Train Station.
- Columns shows the arrival time at each station so the first Glasgow train leaves at 8.20am arriving at Dalmuir at 8.38am, Tarbet at 9.35am, Ardlui at 9.51am and Crianlarich at 10.05am.

Stopwatches

Example 1

A stopwatch shows the following:-

5:31•96

In general, stopwatches show

hours : minutes : seconds . hundredths of a second.

As we can see, no hours have been recorded on this stopwatch so the stopwatch reads:-
5 minutes, 31 seconds and 96 hundredths of a second

Example 2

A stopwatch shows the following:-

This stopwatch shows:-

2 hours, 34 minutes, 53 seconds and 91 hundredths of a second

Using and comparing stopwatches

Here are five times for a race.

The time is recorded in minutes, therefore each stopwatch shows minutes, seconds and milliseconds

5:31•96 5:32•46 5:31•57 5:32•38 5:29•48

We can tell that the slowest time in the race is 5 minutes, 32 seconds and 46 milliseconds.

We can tell that the fastest time in the race is 5 minutes, 29 seconds and 48 milliseconds.

Time 5

Speed, Distance and Time Calculations

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Speed} = \text{Distance} \div \text{Time}$$

$$\text{Time} = \text{Distance} \div \text{Speed}$$

Examples

Distance - Calculate the distance travelled a:-

a) Lorry, going at 30mph for 7 hours

$$D = S \times T$$

$$D = 30 \times 7$$

$$\text{Distance} = 210 \text{ miles}$$

b) plane travelling at 380mph for $4 \frac{1}{2}$ hours

$$D = S \times T \quad D = 380 \times 4 \frac{1}{2}$$

$$\begin{aligned} \text{Distance} &= 380 \times 4 \text{ which is } 1520 \\ &\text{plus } \frac{1}{2} \times 380 \text{ which equals } 190 \text{ miles} \\ \text{Total distance} &= 1710 \text{ miles} \end{aligned}$$

Speed - Work out these average speeds:-

a) A rocket flies 150 metres in 5 seconds

$$S = D \div T$$

$$S = 150 \div 5$$

$$\text{Speed} = 30 \text{ metres per second}$$

b) A plane flies 1820 miles in 5 hours

$$S = D \div T$$

$$S = 1820 \div 5$$

$$\text{Speed} = 364 \text{ miles per hour (mph)}$$

Time - Calculate the time taken for each journey:-

a) driving 490 miles at an average speed of 70mph

$$T = D \div S$$

$$T = 490 \div 70$$

$$\text{Time} = 7 \text{ hours}$$

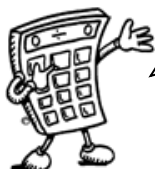
b) skating, at 4 km/hr for 16 km

$$T = D \div S$$

$$T = 16 \div 4$$

$$\text{Time} = 4 \text{ hours}$$

Fractions 1

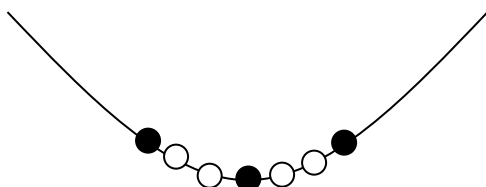


Addition, subtraction, multiplication and division of fractions are studied in mathematics. However, the examples below may be helpful in all subjects.

Understanding Fractions

Example

A necklace is made from black and white beads.



What fraction of the beads are black?

There are 3 black beads out of a total of 7, so $\frac{3}{7}$ of the beads are black.

Equivalent Fractions

Example

What fraction of the flag is shaded?



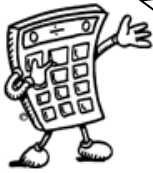
6 out of 12 squares are shaded. So $\frac{6}{12}$ of the flag is shaded.

It could also be said that $\frac{1}{2}$ the flag is shaded.

$\frac{6}{12}$ and $\frac{1}{2}$ are **equivalent fractions**.

Fractions 2

Simplifying Fractions



The top of a fraction is called the **numerator**, the bottom is called the **denominator**.
To simplify a fraction, divide the **numerator** and **denominator** of the fraction by the same number.

Example 1

(a) $\frac{20}{25} = \frac{4}{5}$

Diagram showing the simplification of $\frac{20}{25}$ to $\frac{4}{5}$. A horizontal line with an equals sign in the middle connects the two fractions. Above the line, a curved line connects 20 to 4 with $\div 5$ written above it. Below the line, a curved line connects 25 to 5 with $\div 5$ written below it.

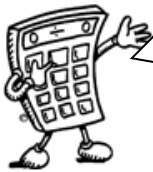
(b) $\frac{16}{24} = \frac{2}{3}$

Diagram showing the simplification of $\frac{16}{24}$ to $\frac{2}{3}$. A horizontal line with an equals sign in the middle connects the two fractions. Above the line, a curved line connects 16 to 2 with $\div 8$ written above it. Below the line, a curved line connects 24 to 3 with $\div 8$ written below it.

This can be done repeatedly until the numerator and denominator are the smallest possible numbers - the fraction is then said to be in its **simplest form**.

Example 2 Simplify $\frac{72}{84}$ $\frac{72}{84} = \frac{36}{42} = \frac{18}{21} = \frac{6}{7}$ (simplest form)

Calculating Fractions of a Quantity



To find the fraction of a quantity, divide by the denominator.

To find $\frac{1}{2}$ divide by 2, to find $\frac{1}{3}$ divide by 3, to find $\frac{1}{7}$ divide by 7 etc.

Example 1 Find $\frac{1}{5}$ of £150

$$\frac{1}{5} \text{ of } \pounds 150 = \pounds 150 \div 5 = \pounds 30$$

Example 2 Find $\frac{3}{4}$ of 48

$$\frac{1}{4} \text{ of } 48 = 48 \div 4 = 12$$

$$\text{so } \frac{3}{4} \text{ of } 48 = 3 \times 12 = 36$$

To find $\frac{3}{4}$ of a quantity, start by finding $\frac{1}{4}$ then multiply by the numerator (3).

Fractions 3

Mixed Numbers and Improper Fractions

- If the numerator of a fraction is smaller than the denominator then the fraction is called a proper fraction.
- If the numerator is larger than the denominator then the fraction is an improper fraction.
- If a number comprises a whole number and a fraction then it is called a mixed number.

Examples

$\frac{3}{4}$ is a proper fraction, $\frac{11}{2}$ is an improper fraction, $2\frac{1}{3}$ is a mixed number.

Conversion Between Mixed Numbers and Improper Fractions

To write an improper fraction as a mixed number we calculate how many whole numbers there are (by considering multiples of the denominator) and then the remaining fraction part.

Examples

$$\begin{aligned} 1. \quad \frac{11}{2} &= \frac{10}{2} + \frac{1}{2} \\ &= 5\frac{1}{2}. \end{aligned}$$

The highest multiple of 2 below 11 is 10 so the whole number must be $\frac{10}{2} = 5$ with $\frac{1}{2}$ left over.

$$\begin{aligned} 2. \quad \frac{14}{3} &= \frac{12}{3} + \frac{2}{3} \\ &= 4\frac{2}{3}. \end{aligned}$$

The highest multiple of 3 below 14 is 12 so the whole number must be $\frac{12}{3} = 4$ with $\frac{2}{3}$ left over.

$$\begin{aligned} 3. \quad \frac{31}{4} &= \frac{28}{4} + \frac{3}{4} \\ &= 7\frac{3}{4}. \end{aligned}$$

The highest multiple of 4 below 31 is 28 so the whole number must be $\frac{28}{4} = 7$ with $\frac{3}{4}$ left over.

Conversely, to write a mixed number as an improper fraction we convert the whole number to a fraction and then add the existing fraction on.

Percentages 1



Percent means out of 100.
A percentage can be converted to an equivalent fraction or decimal.

36% means $\frac{36}{100}$ so could be simplified to $\frac{9}{25}$

36% is equivalent to 0.36

Common Percentages

Some percentages are used very frequently. It is useful to know these as fractions and decimals.

Percentage	Fraction	Decimal
1%	$\frac{1}{100}$	0.01
10%	$\frac{1}{10}$	0.1
20%	$\frac{1}{5}$	0.2
25%	$\frac{1}{4}$	0.25
$33\frac{1}{3}\%$	$\frac{1}{3}$	0.333...
50%	$\frac{1}{2}$	0.5
$66\frac{2}{3}\%$	$\frac{2}{3}$	0.666...
75%	$\frac{3}{4}$	0.75

Percentages 2



There are many ways to calculate percentages of a quantity. Some of the common ways are shown below.

Non- Calculator Methods

Method 1 Using Equivalent Fractions

Example Find 25% of £640

$$25\% \text{ of } \pounds 640 = \frac{1}{4} \text{ of } \pounds 640 = \pounds 640 \div 4 = \pounds 160$$

Method 2 Using 1%

In this method, first find 1% of the quantity (by dividing by 100), then multiply to give the required value.

Example Find 9% of 200g

$$1\% \text{ of } 200\text{g} = \frac{1}{100} \text{ of } 200\text{g} = 200\text{g} \div 100 = 2\text{g}$$

$$\text{so } 9\% \text{ of } 200\text{g} = 9 \times 2\text{g} = 18\text{g}$$

Method 3 Using 10%

This method is similar to the one above. First find 10% (by dividing by 10), then multiply to give the required value.

Example Find 70% of £35

$$10\% \text{ of } \pounds 35 = \frac{1}{10} \text{ of } \pounds 35 = \pounds 35 \div 10 = \pounds 3.50$$

$$\text{so } 70\% \text{ of } \pounds 35 = 7 \times \pounds 3.50 = \pounds 24.50$$

Percentages 3

Non- Calculator Methods (continued)

The previous 2 methods can be combined so as to calculate any percentage.

Example Find 23% of £15000

$$10\% \text{ of } \pounds 15000 = \pounds 1500 \text{ so } 20\% = \pounds 1500 \times 2 = \pounds 3000$$

$$1\% \text{ of } \pounds 15000 = \pounds 150 \text{ so } 3\% = \pounds 150 \times 3 = \pounds 450$$

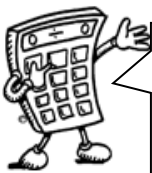
$$23\% \text{ of } \pounds 15000 = \pounds 3000 + \pounds 450 = \pounds 3450$$

Calculator Method

To find the percentage of a quantity using a calculator, change the percentage to a decimal, then multiply.

Example 1 Find 23% of £15000

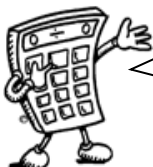
$$23\% = 0.23 \text{ so } 23\% \text{ of } \pounds 15000 = 0.23 \times \pounds 15000 = \pounds 3450$$



We do not use the % button on calculators. The methods taught in the mathematics department are all based on converting percentages to decimals.

Percentages 5

Finding the percentage



To find a percentage of a total, first make a fraction, then convert to a decimal by dividing the top by the bottom. This can then be expressed as a percentage.

Example 1 There are 30 pupils in Class 3A3. 18 are girls.
What percentage of Class 3A3 are girls?

$$\frac{18}{30} = \frac{6}{10} = 60\%$$

Example 2 James scored 36 out of 44 his biology test. What is his percentage mark?

$$\begin{aligned} \text{Score} &= \frac{36}{44} = 36 \div 44 = 0.81818\dots \\ &= 81.818\dots\% = 82\% \text{ (rounded)} \end{aligned}$$

Example 3 In class 1X1, 14 pupils had brown hair, 6 pupils had blonde hair, 3 had black hair and 2 had red hair. What percentage of the pupils were blonde?

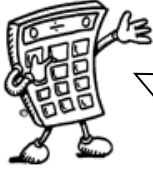
$$\text{Total number of pupils} = 14 + 6 + 3 + 2 = 25$$

6 out of 25 were blonde, so,

$$\frac{6}{25} = \frac{24}{100} = 24\%$$

24% were blonde.

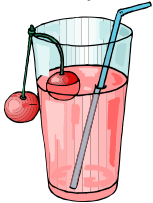
Ratio 1



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

Writing Ratios

Example 1



To make a fruit drink, 4 parts water is mixed with 1 part of cordial.

The ratio of water to cordial is 4:1

(said "4 to 1")

The ratio of cordial to water is 1:4.

Order is important when writing ratios.

Example 2



In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.

The ratio of red : blue : green is 5 : 7 : 8

Using ratios

The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

Fruit	Nuts
3	2
x5 15	x5 10

So the chocolate bar will contain 10g of nuts.

Proportion



Two quantities are said to be in direct proportion if when one doubles the other doubles.
We can use proportion to solve problems.

It is often useful to make a table when solving problems involving proportion.

Example 1

A car factory produces 1500 cars in 30 days. How many cars would they produce in 90 days?

Days	Cars
30	1500
90	4500

$\times 3$ (under 30 to 90) and $\times 3$ (under 1500 to 4500)

The factory would produce 4500 cars in 90 days.

Information Handling : Tables



It is sometimes useful to display information in graphs, charts or tables.

Example 1 The table below shows the average maximum temperatures (in degrees Celsius) in Barcelona and Edinburgh.

	J	F	M	A	M	J	J	A	S	O	N	D
Barcelona	13	14	15	17	20	24	27	27	25	21	16	14
Edinburgh	6	6	8	11	14	17	18	18	16	13	8	6

The average temperature in June in Barcelona is 24°C

Frequency Tables are used to present information. Often data is grouped in intervals.

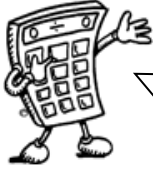
Example 2 Homework marks for Class 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
 33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

Mark	Tally	Frequency
16 - 20		2
21 - 25		7
26 - 30		9
31 - 35		5
36 - 40		3
41 - 45		2
46 - 50		2

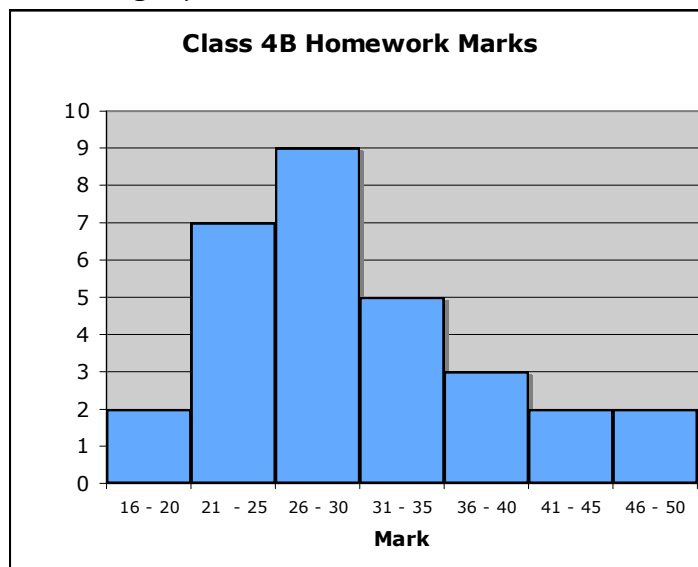
Each mark is recorded in the table by a tally mark. Tally marks are grouped in 5's to make them easier to read and count.

Information Handling : Bar Graphs

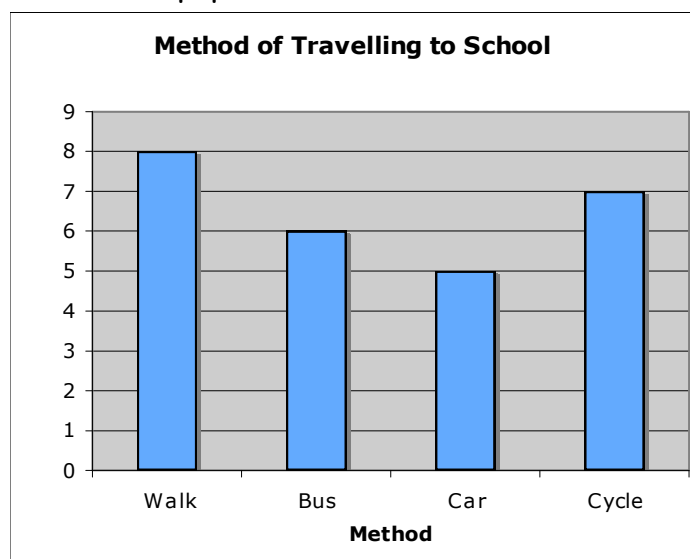


Bar graphs are often used to display data. The horizontal axis should show the categories or class intervals, and the vertical axis the frequency. All graphs should have a title, and each axis must be labelled.

Example 1 The graph below shows the homework marks for Class 4B.

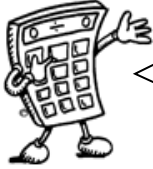


Example 2 How do pupils travel to school?



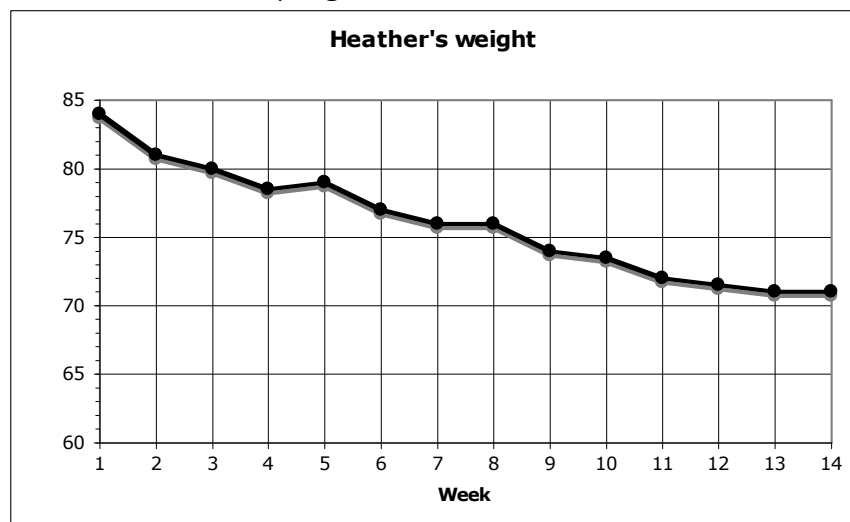
When the horizontal axis shows categories, rather than grouped intervals, it is common practice to leave gaps between the bars.

Information Handling : Line Graphs



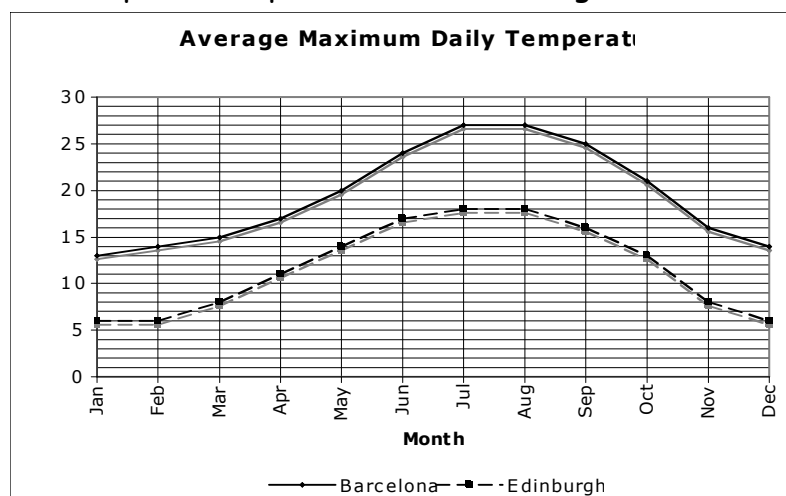
Line graphs consist of a series of points which are plotted, then joined by a line. All graphs should have a title, and each axis must be labelled. The trend of a graph is a general description of it.

Example 1 The graph below shows Heather's weight over 14 weeks as she follows an exercise programme.

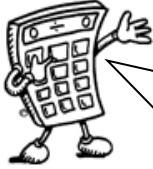


The trend of the graph is that her weight is decreasing.

Example 2 Graph of temperatures in Edinburgh and Barcelona.



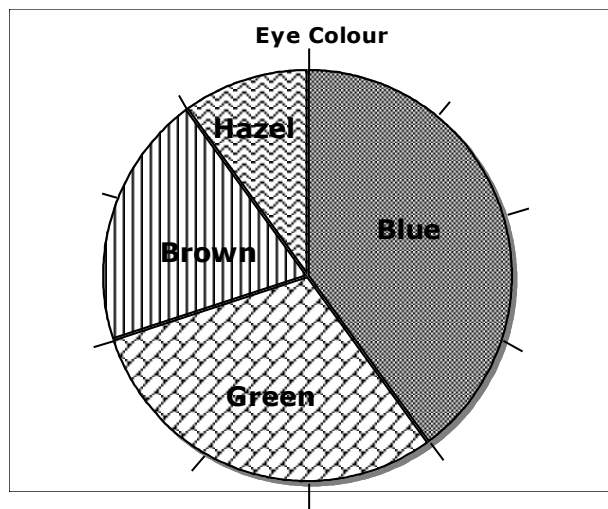
Information Handling : Pie Charts



A pie chart can be used to display information. Each sector (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

Example

30 pupils were asked the colour of their eyes. The results are shown in the pie chart below.

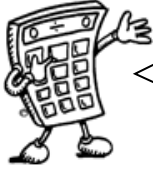


How many pupils had brown eyes?

The pie chart is divided up into ten parts, so pupils with brown eyes represent $\frac{2}{10}$ of the total.

$\frac{2}{10}$ of 30 = 6 so 6 pupils had brown eyes.

Information Handling : Averages



To provide information about a set of data, the average value may be given. There are 3 ways of finding the average value - the mean, the median and the mode.

Mean

The mean is found by adding all the data together and dividing by the number of values.

Median

The median is the middle value when all the data is written in numerical order (if there are two middle values, the median is half-way between these values).

Mode

The mode is the value that occurs most often.

Range

The range of a set of data is a measure of spread.

Range = Highest value - Lowest value

Example Class 1A4 scored the following marks for their homework assignment. Find the mean, median, mode and range of the results.

7, 9, 7, 5, 6, 7, 10, 9, 8, 4, 8, 5, 7, 10

$$\begin{aligned}\text{Mean} &= \frac{7+9+7+5+6+7+10+9+8+4+8+5+7+10}{14} \\ &= \frac{102}{14} = 7.285\dots \quad \text{Mean} = 7.3 \text{ to 1 decimal place}\end{aligned}$$

Ordered values: 4, 5, 5, 6, 7, 7, 7, 7, 8, 8, 9, 9, 10, 10
Median = 7

7 is the most frequent mark, so Mode = 7

Range = 10 - 4 = 6

Mathematical Dictionary (Key words):

Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) Example: $12+76 = 88$
a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon).
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Data	A collection of information (may include facts, numbers or measurements).
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction). Example: The difference between 50 and 36 is 14 $50 - 36 = 14$
Division (\div)	Sharing a number into equal parts. $24 \div 6 = 4$
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Equivalent fractions	Fractions which have the same value. Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Greater than ($>$)	Is bigger or more than. Example: 10 is greater than 6. $10 > 6$
Least	The lowest number in a group (minimum).
Less than ($<$)	Is smaller or lower than. Example: 15 is less than 21. $15 < 21$.

Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers (see p32)
Median	Another type of average - the middle number of an ordered set of data (see p32)
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.
Mode	Another type of average - the most frequent number or category (see p32)
Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72
Multiply (x)	To combine an amount a particular number of times. Example $6 \times 4 = 24$
Negative Number	A number less than zero. Shown by a minus sign. Example -5 is a negative number.
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1, 3, 5, 7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done. BODMAS (see p9)
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight).
Prime Number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor.
Product	The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20.
Remainder	The amount left over when dividing a number.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Total	The sum of a group of numbers (found by adding).